Dynamics of Transient Liquid Injection:

K-H instability, vorticity dynamics, R-T instability, capillary action, and cavitation

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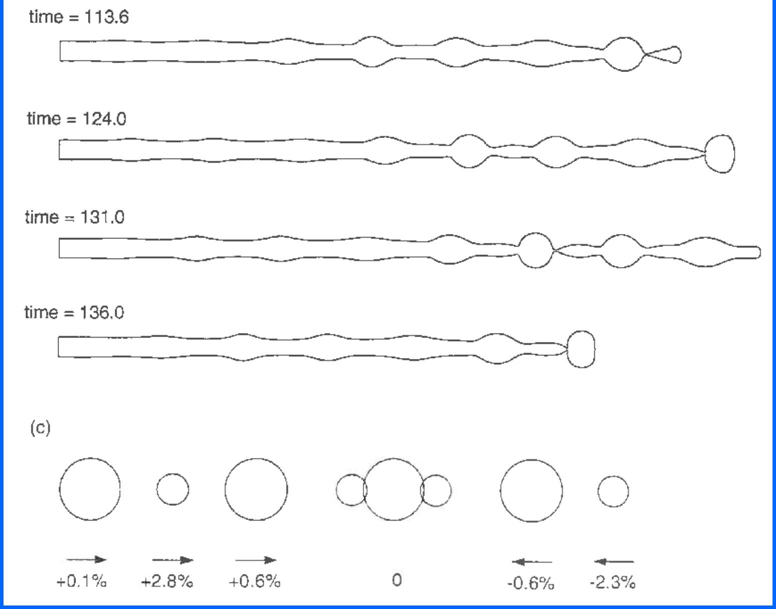
-- Round liquid columns without aerodynamic effects
-- Stretched "conical" and fan jets without aerodynamic effects
-- Planar free films with aerodynamic effects
-- Round jets without surface tension and density discontinuity
-- Round jets with surface tension and density discontinuity
-- Liquid segment analysis
-- Cavitation and bubble growth, distortion, and collapse

Support from US Army Research Office
Collaborations: former students, Dr C. Mehring and Dr. S. Dabiri; current student, Ms. D. Jarrahbashi; Prof. R. H. Rangel, and Prof. D. D. Joseph.

Stability of Cylindrical Liquid Column: Rayleigh Analysis



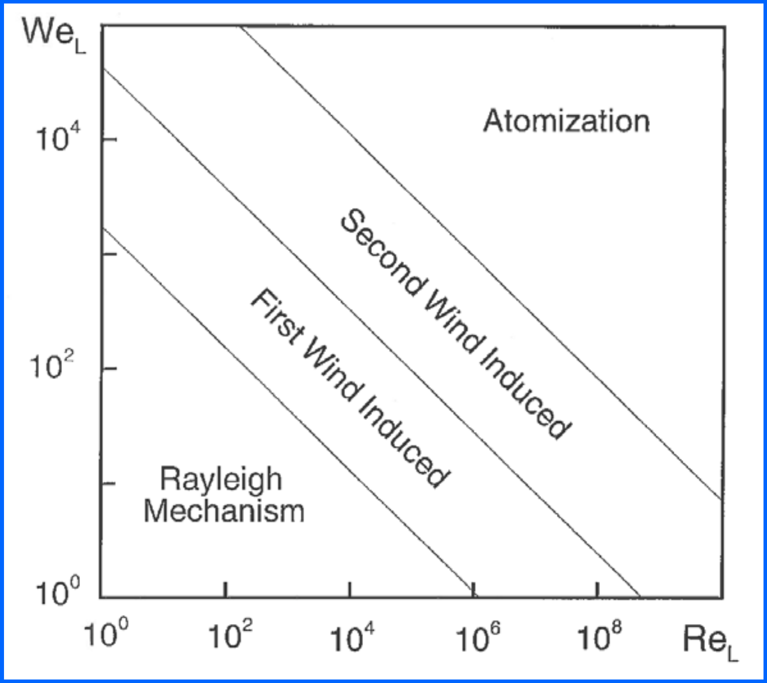
Surface area and surface energy of resulting drops are 79% of original cylinder values. Internal liquid pressure increases by about 6% causing a few percent increase in liquid enthalpy. Remaining enthalpy increase occurs due to viscous dissipation.

$$\frac{S_{drop}}{S_0} = 2\left(\frac{R_{drop}}{R_0}\right)^2 \left(\frac{\lambda}{R_0}\right)^{-1} = 0.7933$$

$$\frac{2R_0}{R_{drop}} = 2\left(\frac{4}{3}\frac{R_0}{\lambda}\right)^{1/3} = 1.058$$

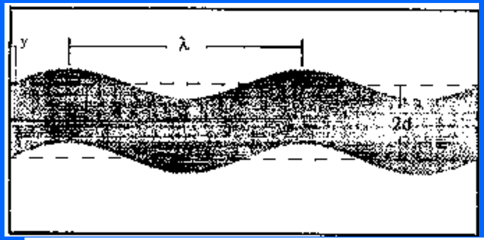
A more accurate nonlinear analysis predicts small satellite droplets also occur.

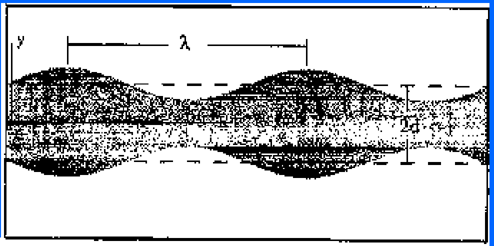
To obtain many small droplets (i.e., spray) and increase surface energy, kinetic energy of surrounding stream must be exchanged.

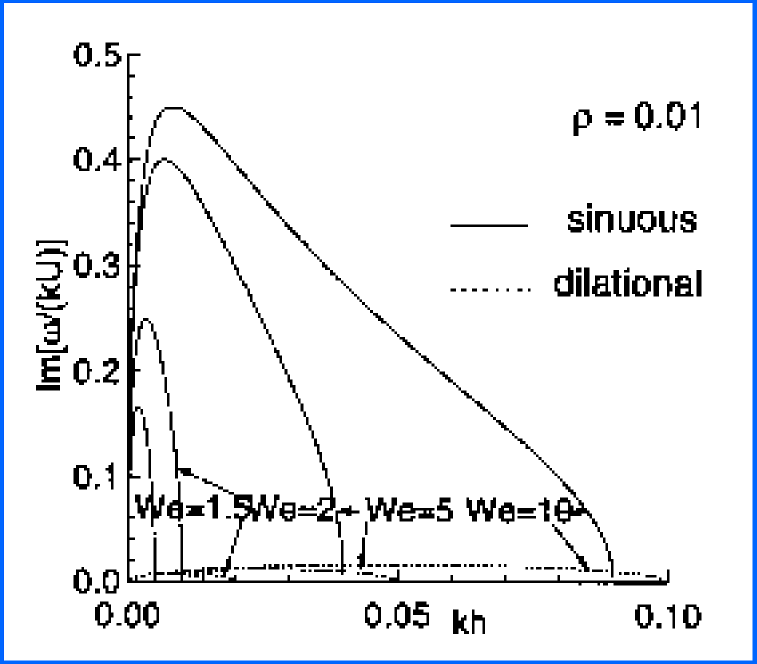


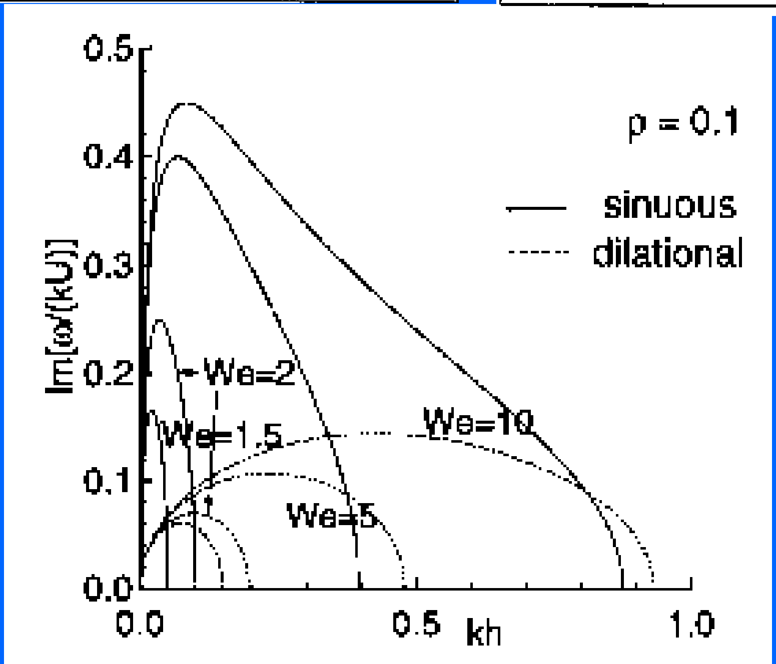
Capillary / Kelvin-Helmholtz instability of planar liquid sheets

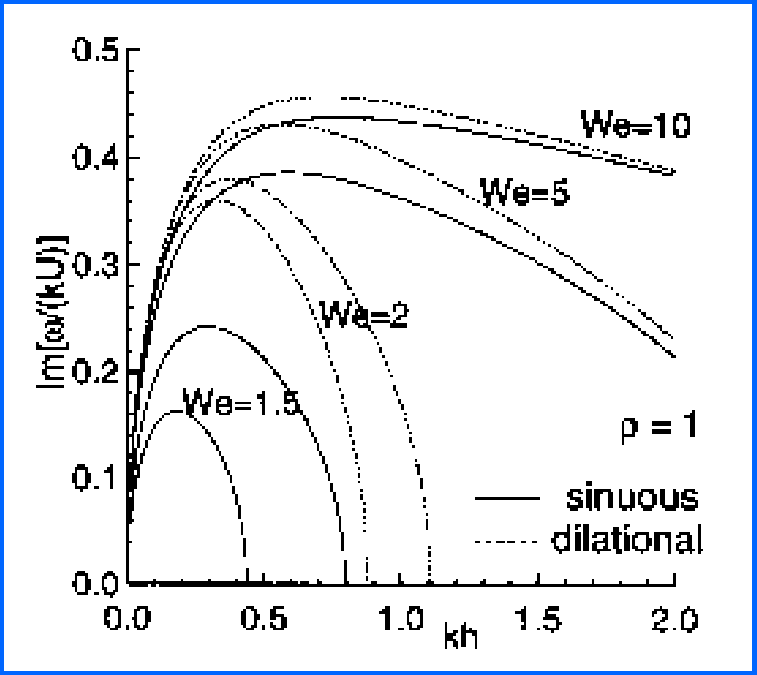
Linear theory predicts the dilational mode becomes more probable than the sinuous mode at high pressures











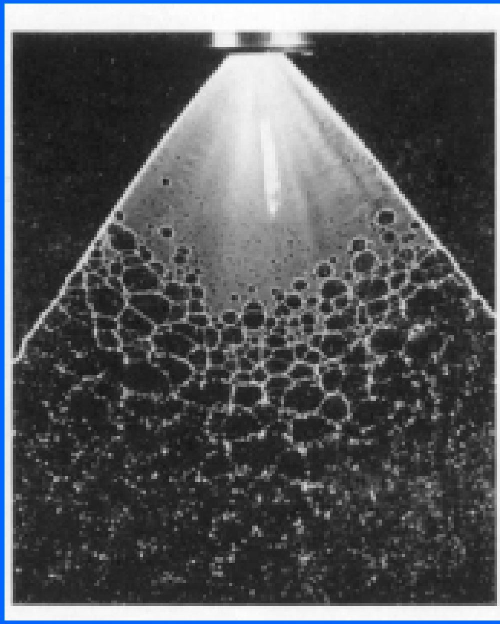
Capillary Action on Thin Sheets: e.g., fan jet and "conical" jet

Dilational oscillations of thin free liquid films or sheets can result in local tearing.

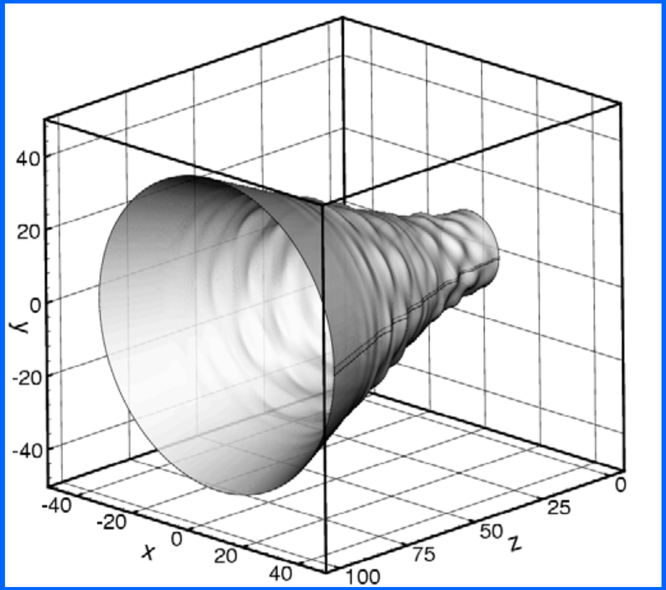
No aerodynamic effects here.

Linear theory predicts dilational oscillations are more likely at high pressure.

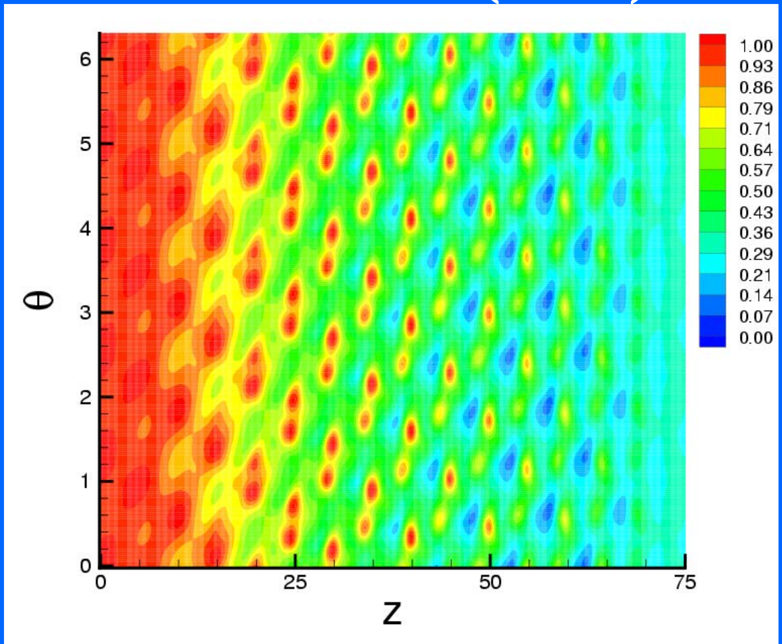
Tearing of thin lobes formed in secondary instabilities will be addressed later.



Dombrowski & Fraser (1954)



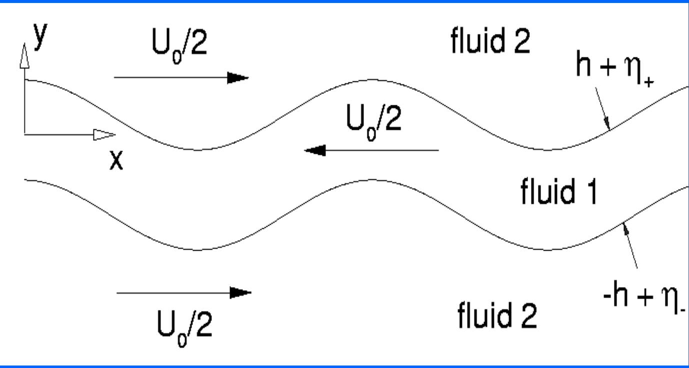
Mehring & Sirignano (2004)



Vortex Dynamics at Liquid-Gas Interface in Planar Flow

The phase interface is a vortex sheet with velocity discontinuity. Examine nonlinear Kelvin-Hemholtz instability with surface tension and density jump.

Approximate sheet as a packed linear array of "vortex blobs."



$$\frac{d(\Delta\Gamma)}{dt} = 2A\left(\frac{du}{dt}\right)_s \Delta s + \frac{A}{4}\frac{\partial}{\partial s}\left(\frac{\Delta\Gamma}{\Delta s}\right)^2 \Delta s - \frac{2}{\mathrm{We}}\frac{\partial \kappa}{\partial s}\Delta s,$$

$A = (\rho_2 - \rho_1)/(\rho_2 + \rho_1)$; $We = (\rho_2 + \rho_1)\,\lambda(\Delta U)^2/\sigma$

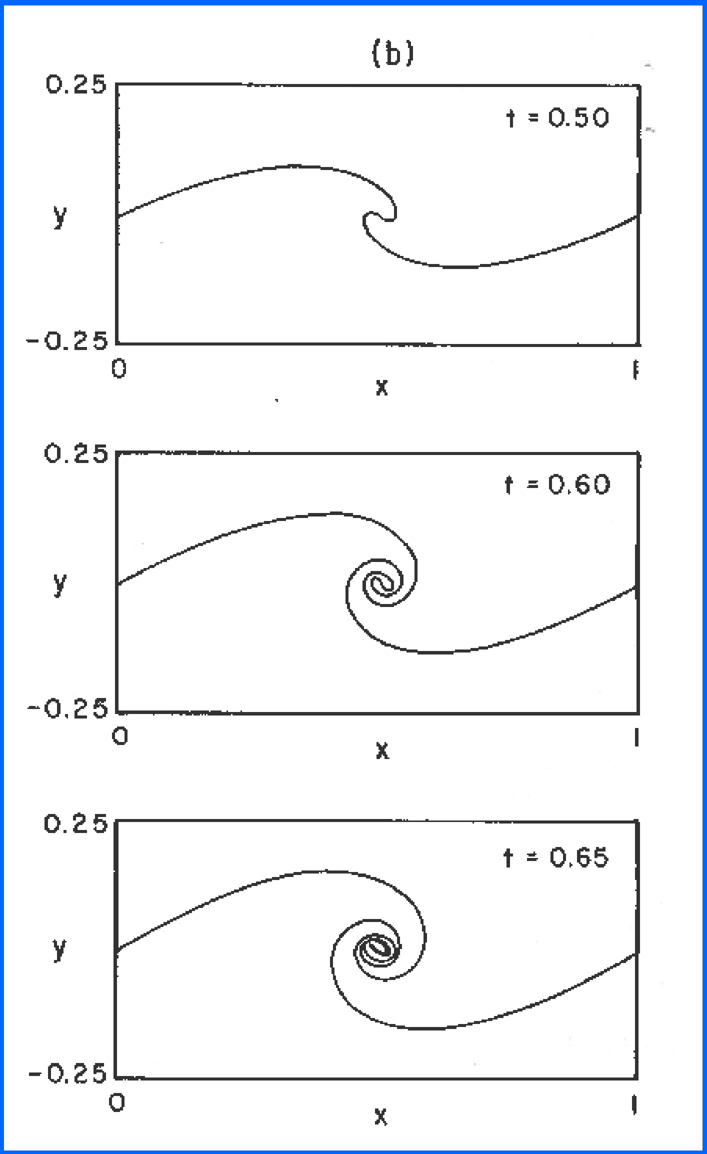
Zalosh (1976) and Rottman & Olfe (1977)

Circulation changes with time due to surface tension.

Also, density difference causes a temporal change in circulation.

Similar qualitative behavior might be expected for axisymmetric flow.

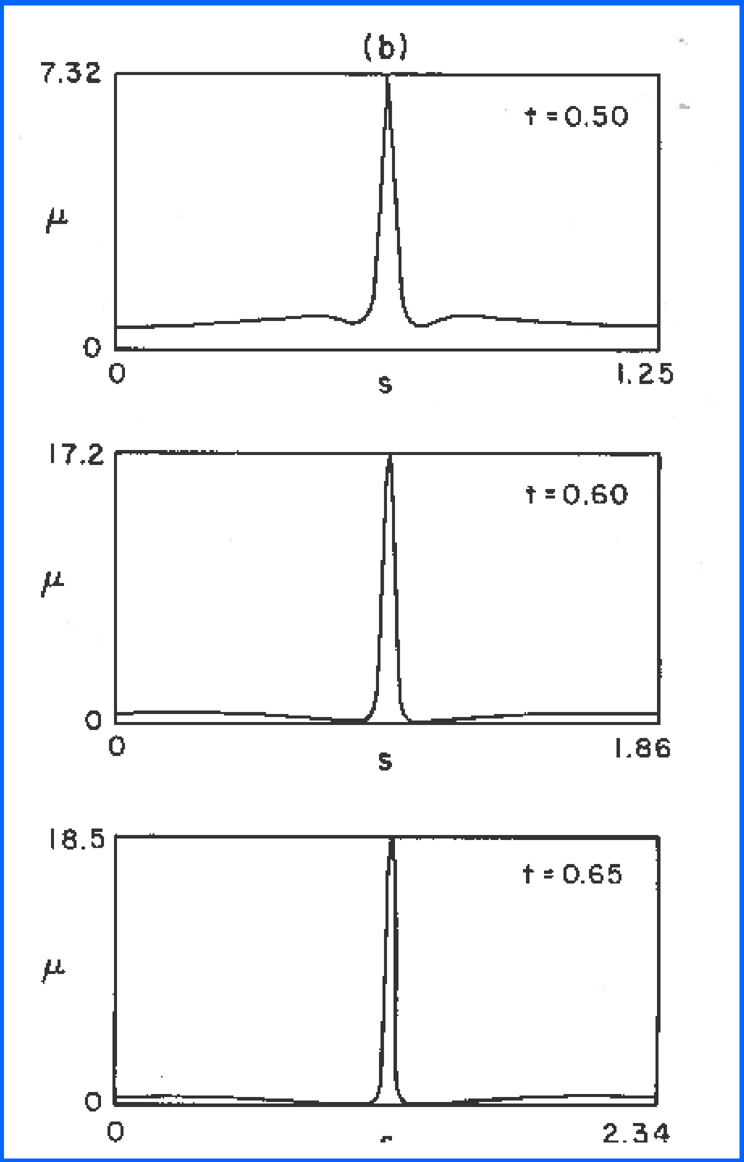
Surface contour and vorticity with no surface tension and no density difference



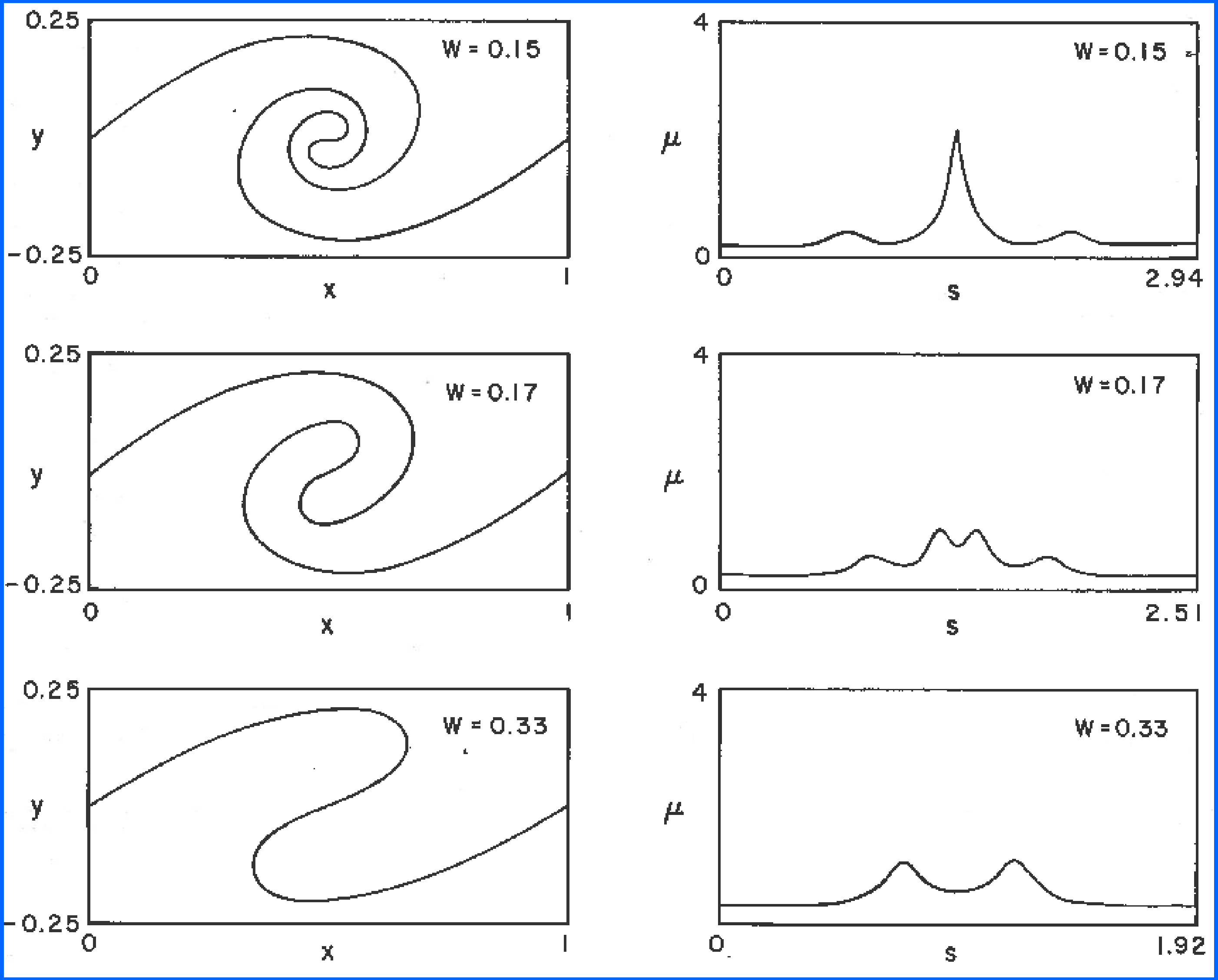
← Contour

Vorticity →

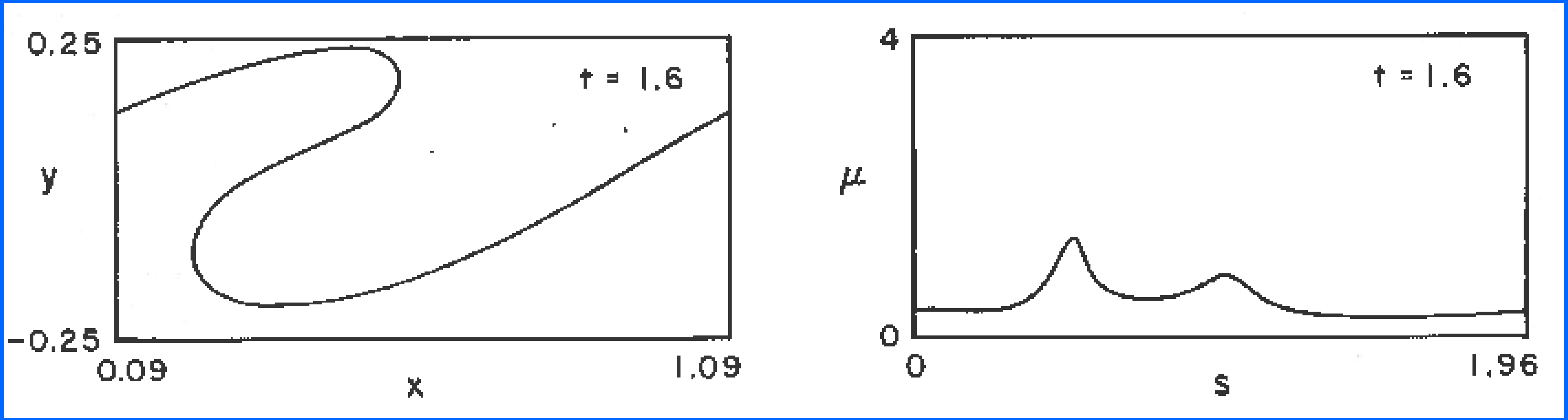
Rangel & Sirignano (1988)



Surface tension inhibits roll-up and vorticity concentration: no density difference



Density reduction also inhibits roll-up, here in combination with surface tension



-- "Cone" shape surface results from surface tension and density discontinuity.
-- Similar shape distortion from KH instability with surface tension and density discontinuity is seen for axisymmetric case, i.e., round jet.
-- 3D distortions on these cones can be expected.

Planar free film with K-H instability and capillary action. A cone shape can develop in time with concentration of vorticity.

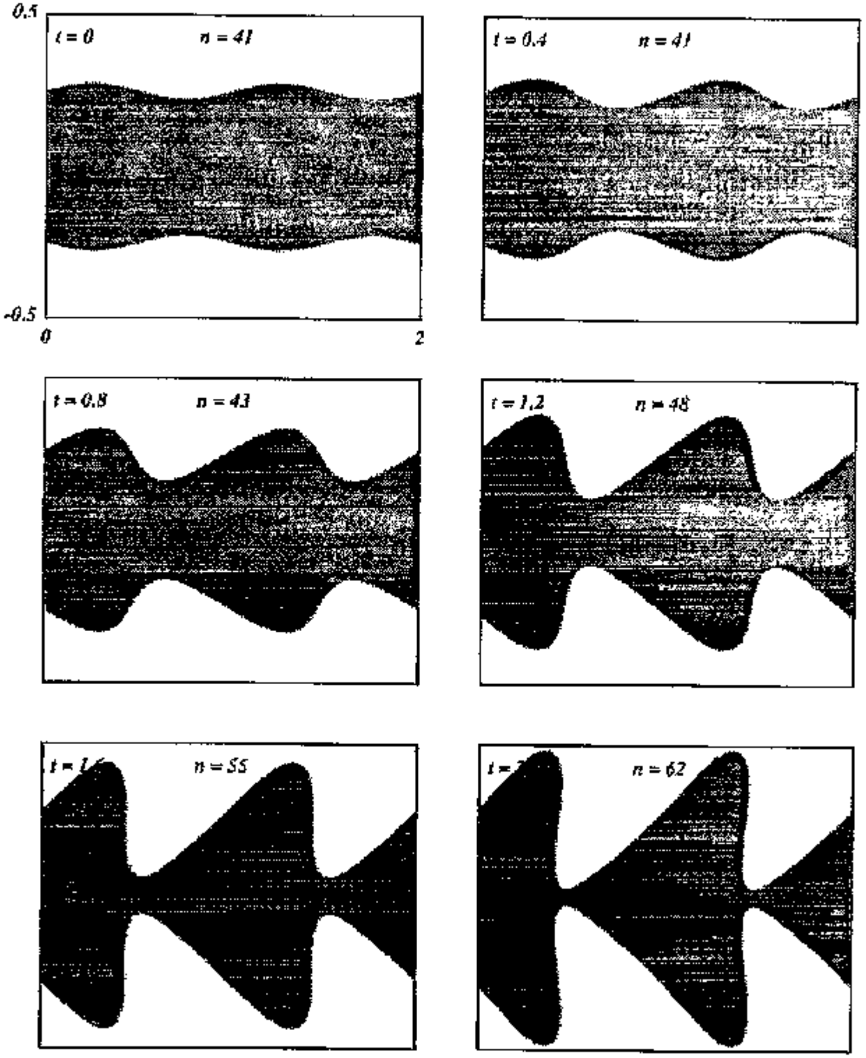


FIG. 14. Time evolution of the dilational mode for $\rho = 1$, $W = 0.67$, and $h = 0.25$ ($H = 0.373$).

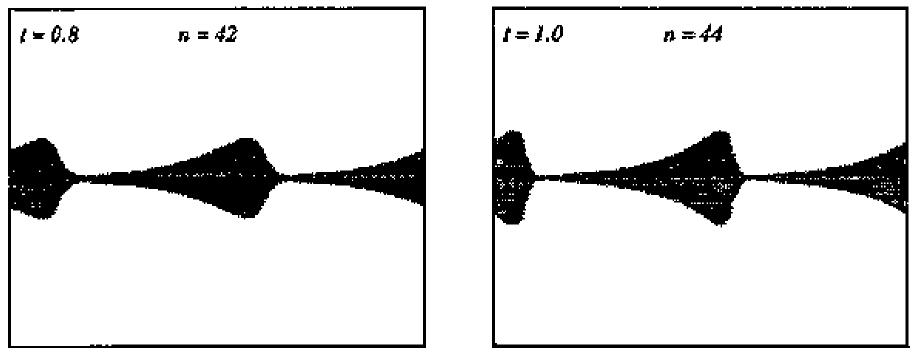


FIG. 8. Time evolution of the dilational mode for $\rho = 0.25$, $W = 0.5$, and $h = 0.05$ ($H = 0.1$).

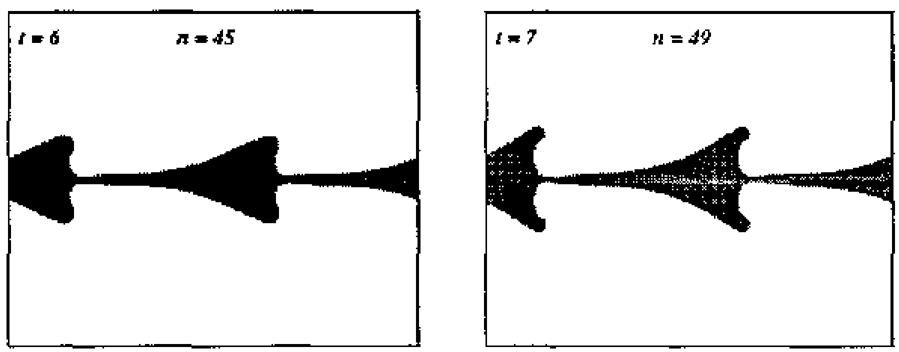
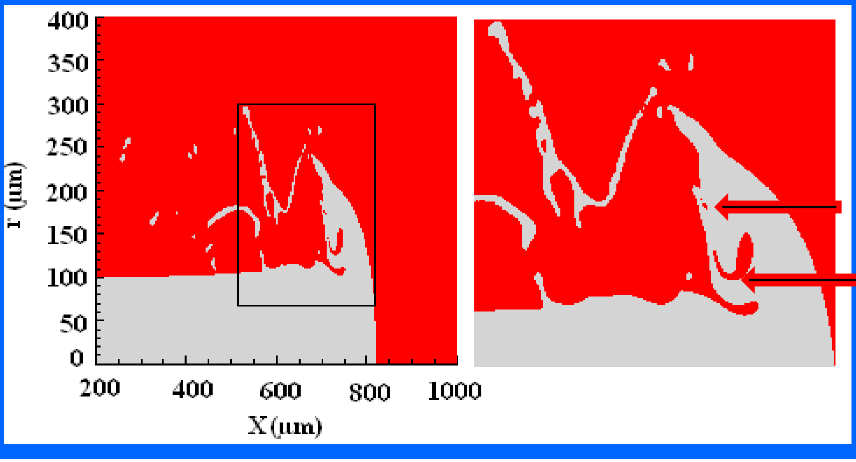


FIG. 11. Time evolution of the dilational mode for $\rho = 0.25$, $W = 0.05$, and $h = 0.05$ ($H = 0.1$).

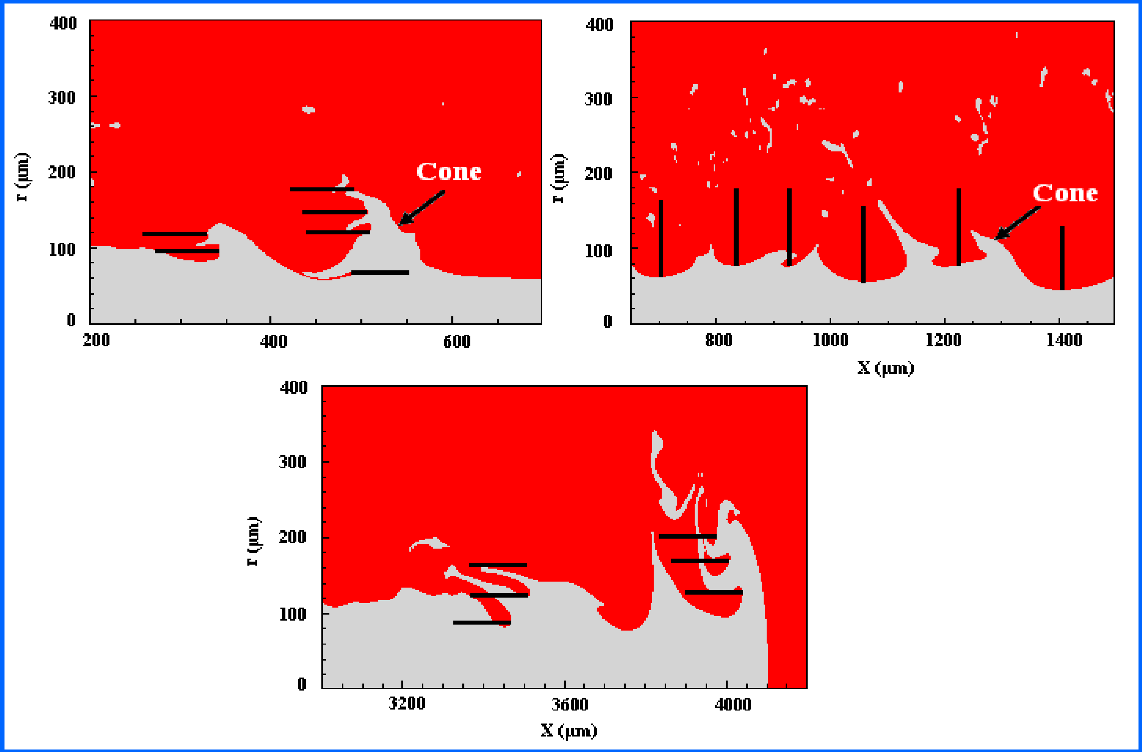
Rangel & Sirignano (1991)

Axisymmetric round jet with K-H and R-T instabilities



Re=16,000, We=230,000, t=40 μs

- Start-up: Heavier fluid on cap accelerates into the lighter fluid; R-T instability on the rear side of the jet cap.
- R-T instability appear on top of the K-H cones drawn out of the liquid.



Re=16,000,
We=230,000,
t=100 μs

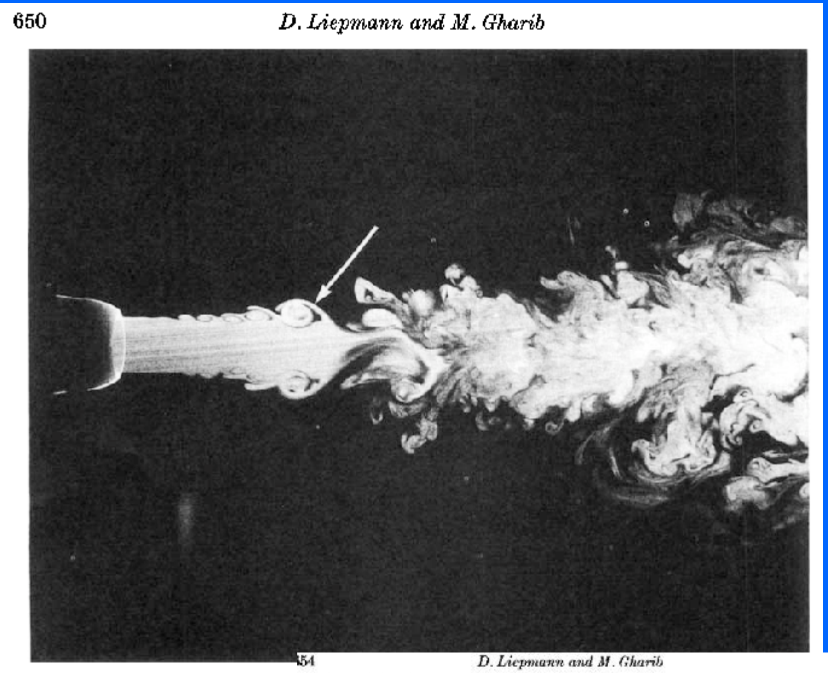
Jarrahbashi & Sirignano (2013)

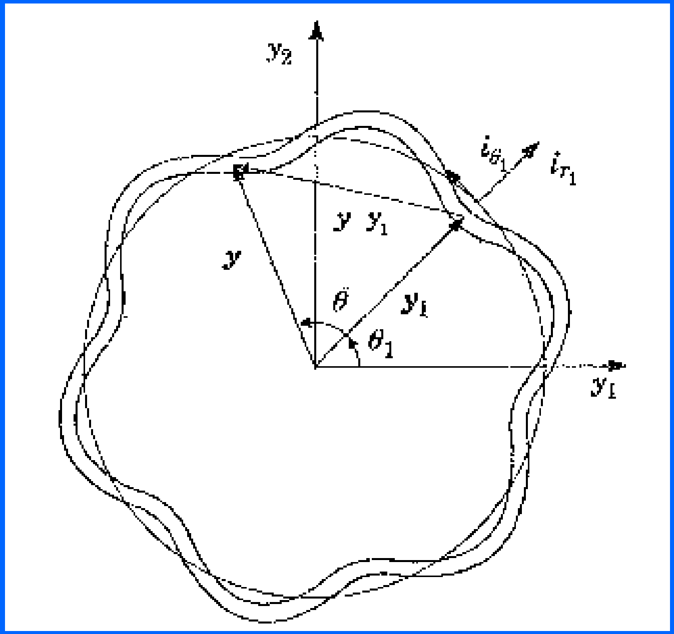
Round jets and unstable vortex rings

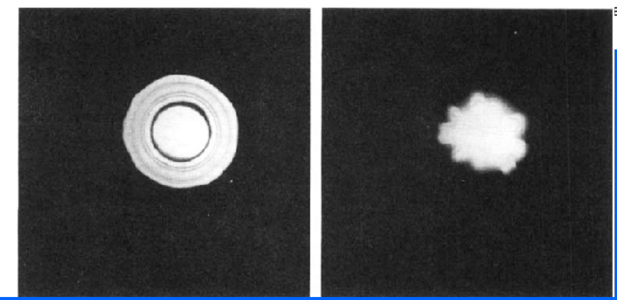
Axisymmetric vortex rings develop 3D instabilities

Liepmann& Gharib (1992), water-into-water

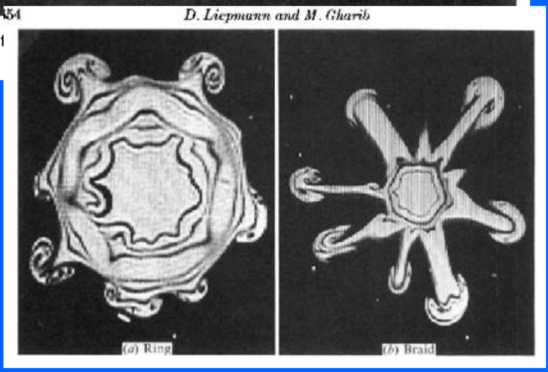
Widnall &Sullivan (1973), air-into-air



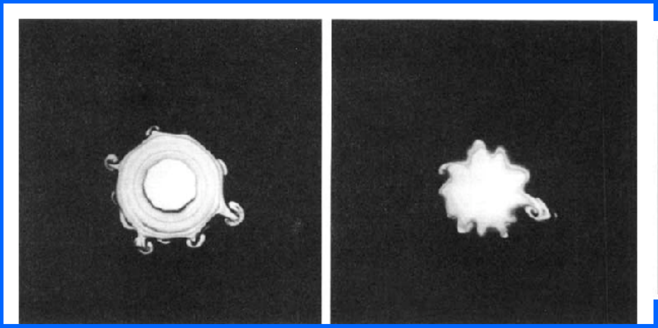


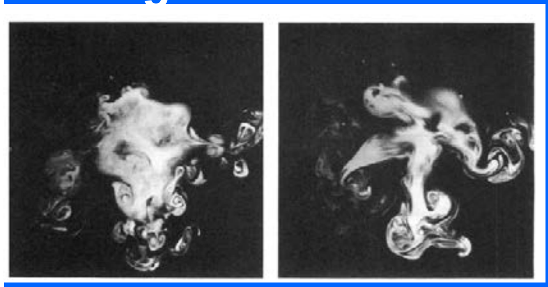


Ring **Braid**



Ring **Braid**





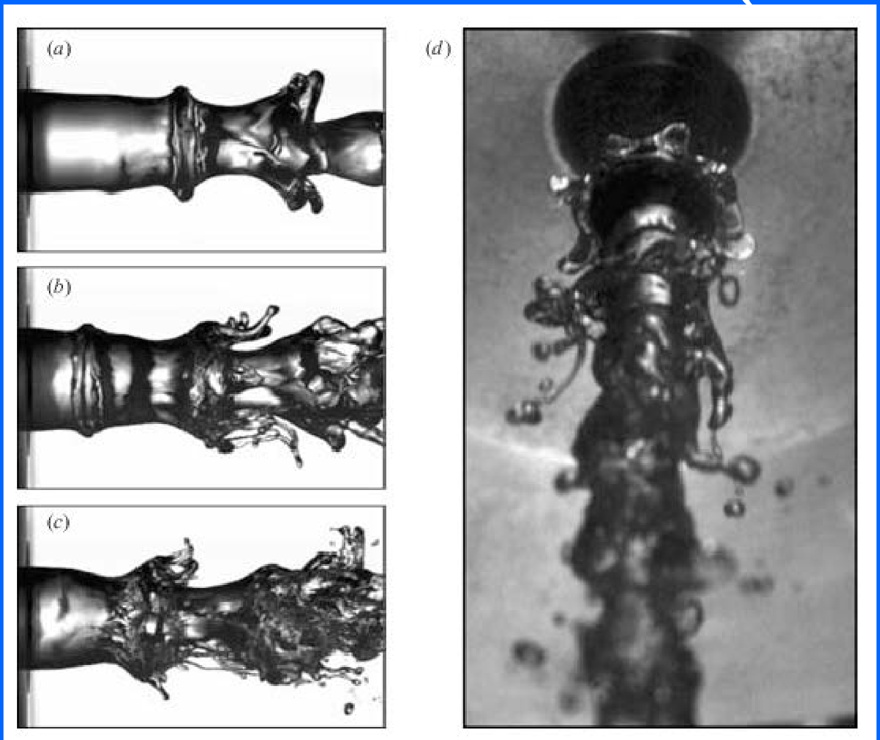
Liquid jet with co-axial air

-- Cones form.

-- Azimuthal lobes form on cones with attribution to Rayleigh-Taylor instability.

-- Ligaments form from lobes.

Marmottant & Villermaux (2004)



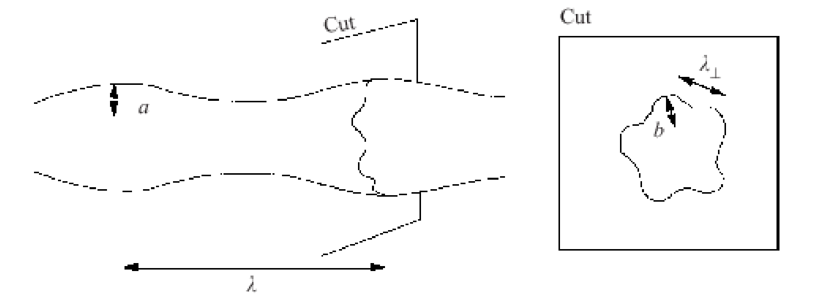


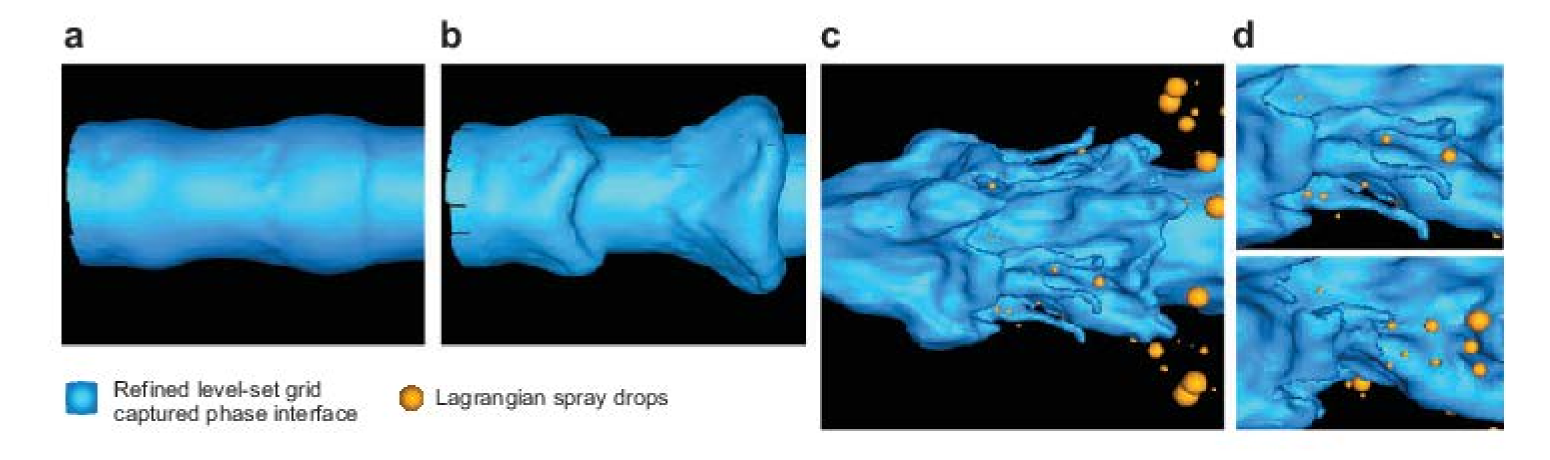
FIGURE 24. Development of the transverse instability.

$$\omega_{i\,RT} = \left(\frac{2}{3\sqrt{3}}\right)^{1/2} \left(\frac{\rho_1 g^3}{\sigma}\right)^{1/4},$$

$$\lambda_{RT} = 2\pi \left(\frac{3\sigma}{\rho_1 g}\right)^{1/2},$$

How does vorticity dynamics act in lobe formation?

3D Navier-Stokes / Level-Set Solutions



Re_g = 3770
We_g = 34
Re_l = 295
We_l = 0.6

Kim, Desjardins, Herrmann, and Moin (2007) – liquid jet with co-axial air flow, simulated M & V experiment.

-- K-H axisymmetric waves deform to cone shape;

-- Azimuthal distortion (attributed to R-T instability) yields lobes, then ligaments;

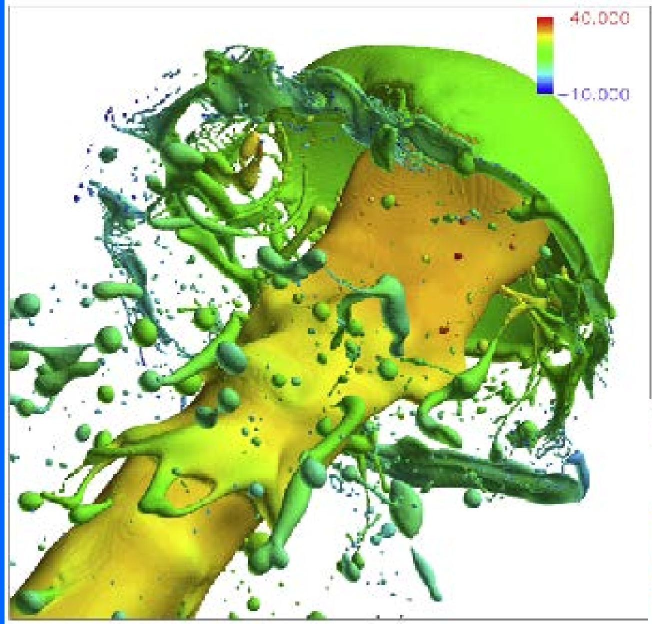
-- Ligaments pinch-off via capillary action to yield droplets.

Does 3D vorticity dynamics affect lobe formation? Are the lobes and ligaments "cousins" of the single-phase vortex-pairs? Does R-T instability occur on ligaments also?

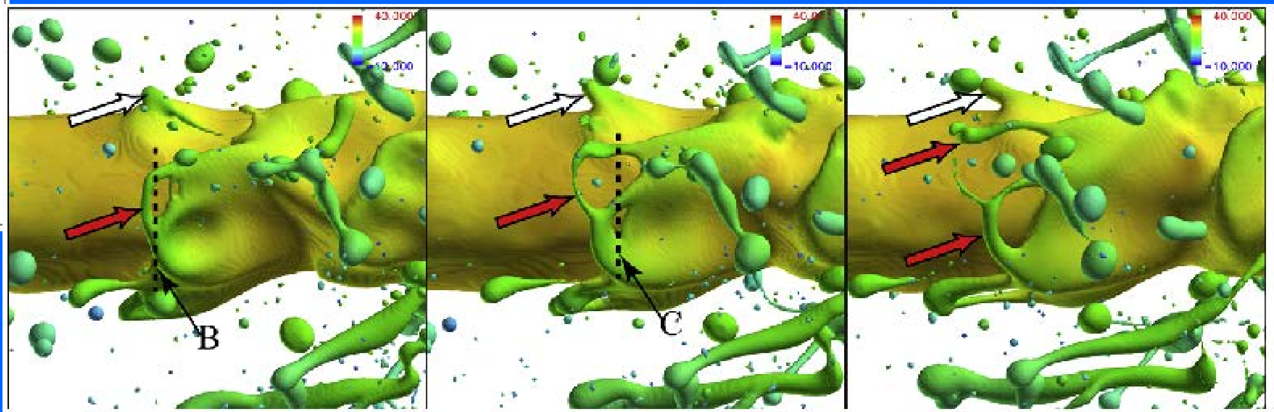
Liquid transient, round jet into still air

Shinjo & Umemura (2010), NS-LS

Re = 440 – 1470; We = 1270 – 14,100



-- Cap forms on starting jet.
-- Mass and size of cap increase.
-- Instability causes shredding of cap.
-- Lobes form on jet stem or core surface.

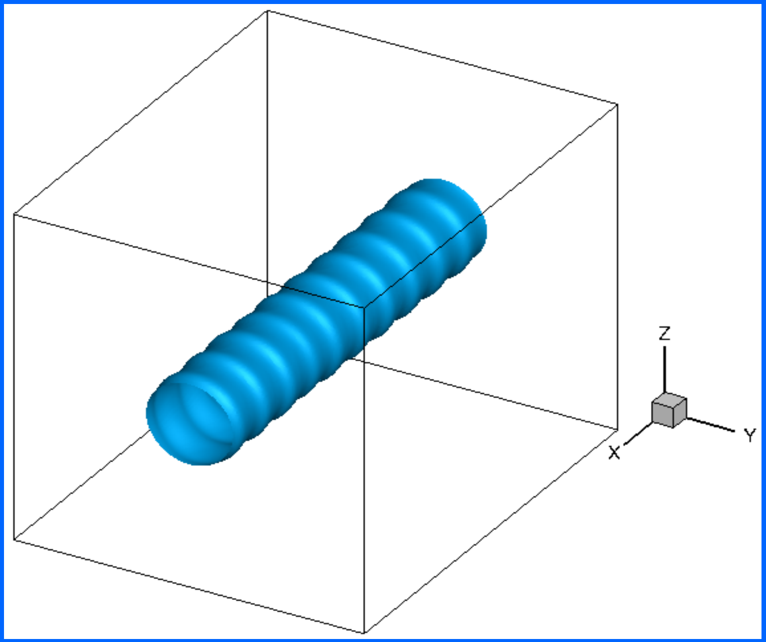


-- Cones have azimuthal instability. Lobes form.
-- Lobes stretch and tear under azimuthal instability.
-- Ligaments form from torn rims.
-- Capillary instabilities on ligaments. (*What about R-T instability on ligament?)*
-- Droplets impact core surface.

Three-dimensional Liquid Jet Segment
N-S / L-S solutions

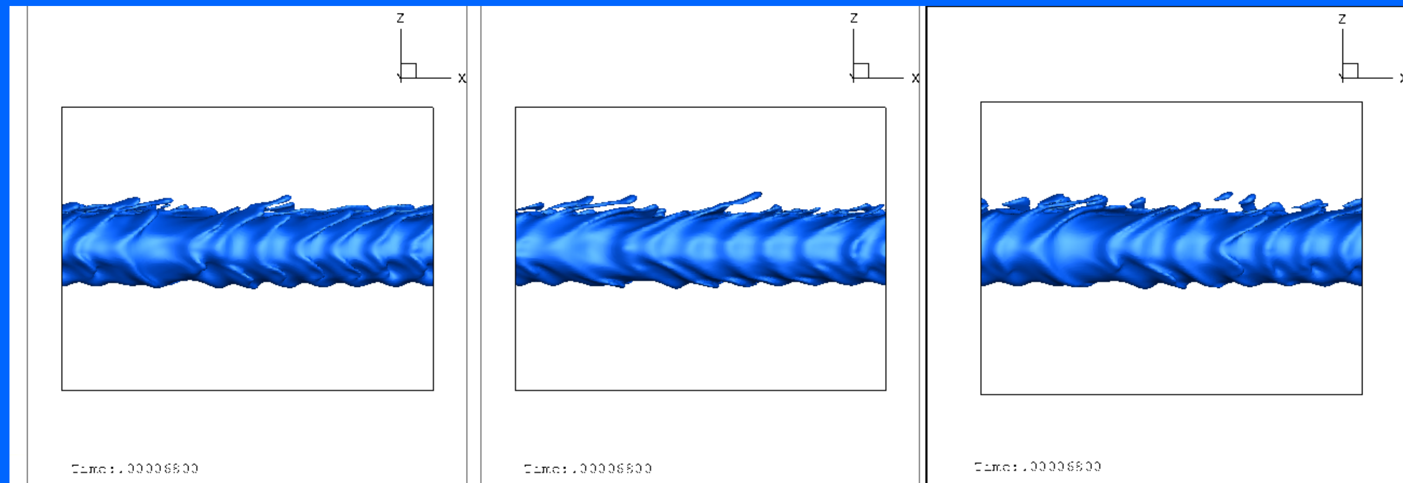
Jarrahbashi & Sirignano (2013)

- **The liquid-segment with 1 mm length**
- **and initial 200 μm diameter.**
- **Continuity of the velocity and shear stress across the interface.**
- **Periodic boundary condition for all three components of velocity in the x-direction.**
- **Zero normal gradients of velocity and pressure on the outer *xz* and *xy* planes**
- **Initial disturbance is a sinusoidal surface wave with wavelengths 100 μm.**
- **The initial wavelength was chosen to be consistent with full-jet results.**
- **Vortex rings form from the primary shear (K-H) instability at the jet interface.**



Asymmetric 3D Distortion is not strongly dependent on initial conditions

$t = 67\ \mu s$



Re = 1600

We = 230,000

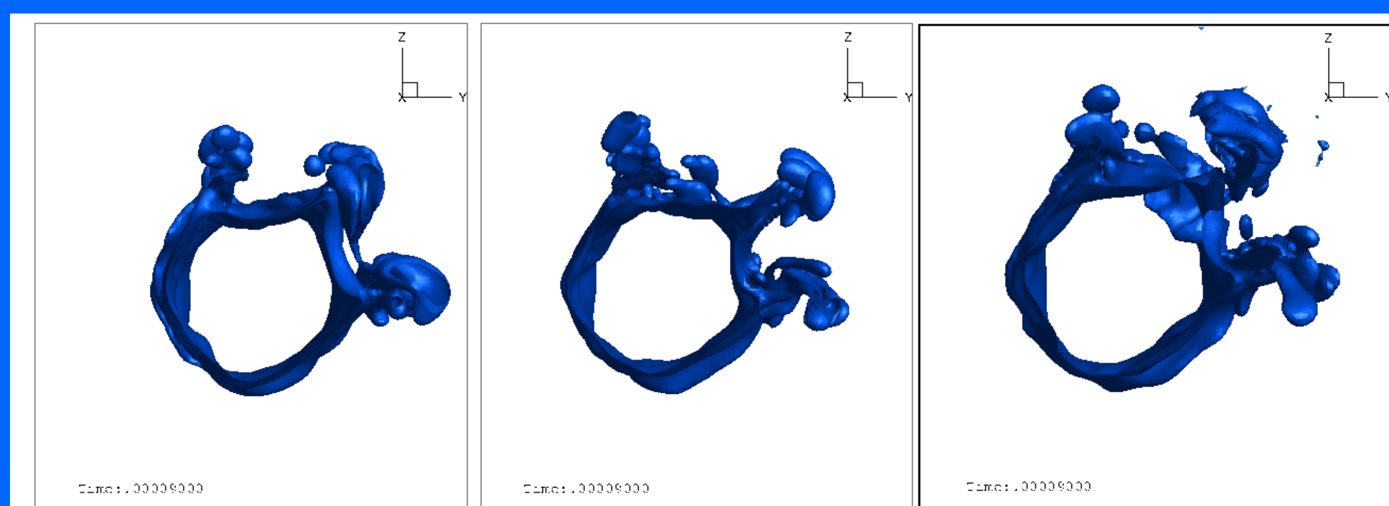
$\rho_1 / \rho_2 = 0.1$

$\mu_1 / \mu_2 = 9 \times 10^{-4}$

Sin 3θ | *Sin 5θ* | *Axisymmetric*

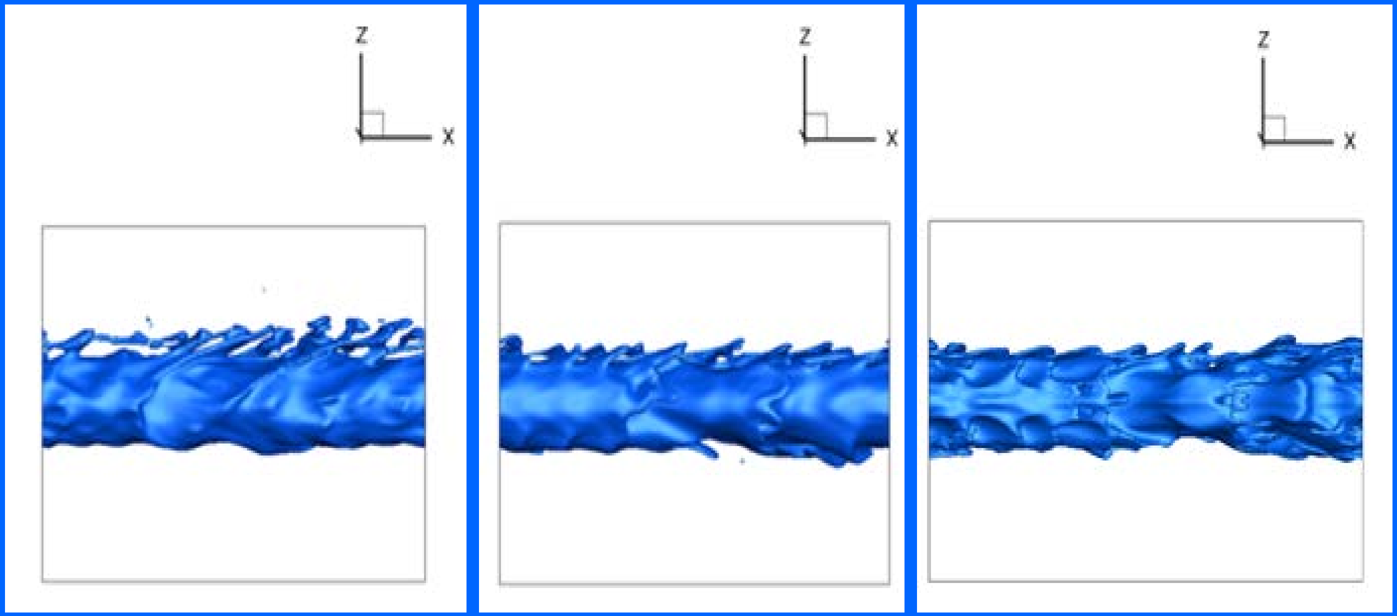
Initial perturbation

$t = 90\ \mu s$

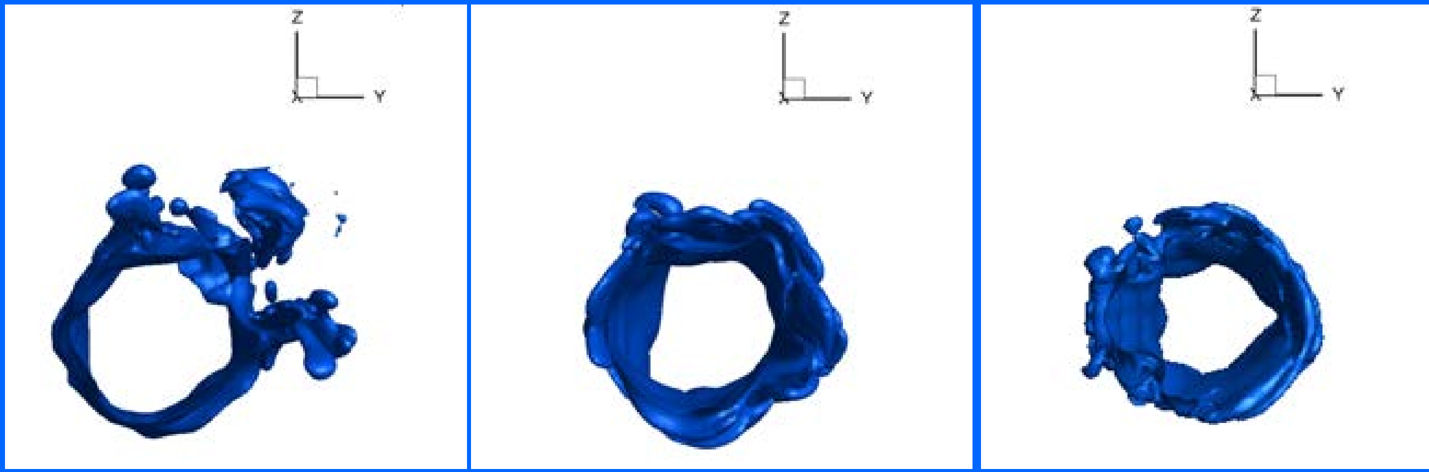


Density Ratio affects 3D instability

***Re* = 1600**
***We* = 230,000**
$\mu_1 / \mu_2 = 9 \times 10^{-4}$
$t = 90\ \mu s$



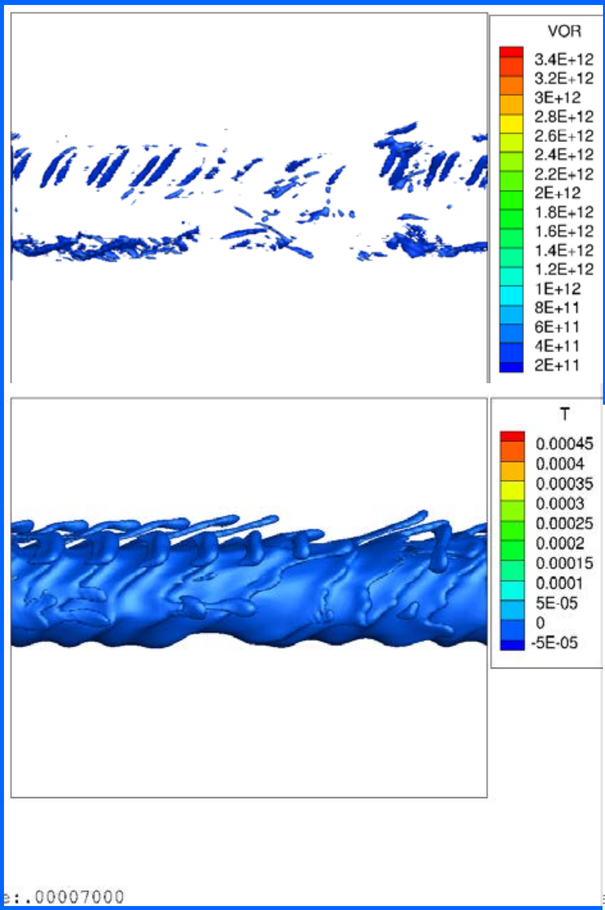
$\rho_1 / \rho_2 =$ **0.1 , 0.5 , 0.9**

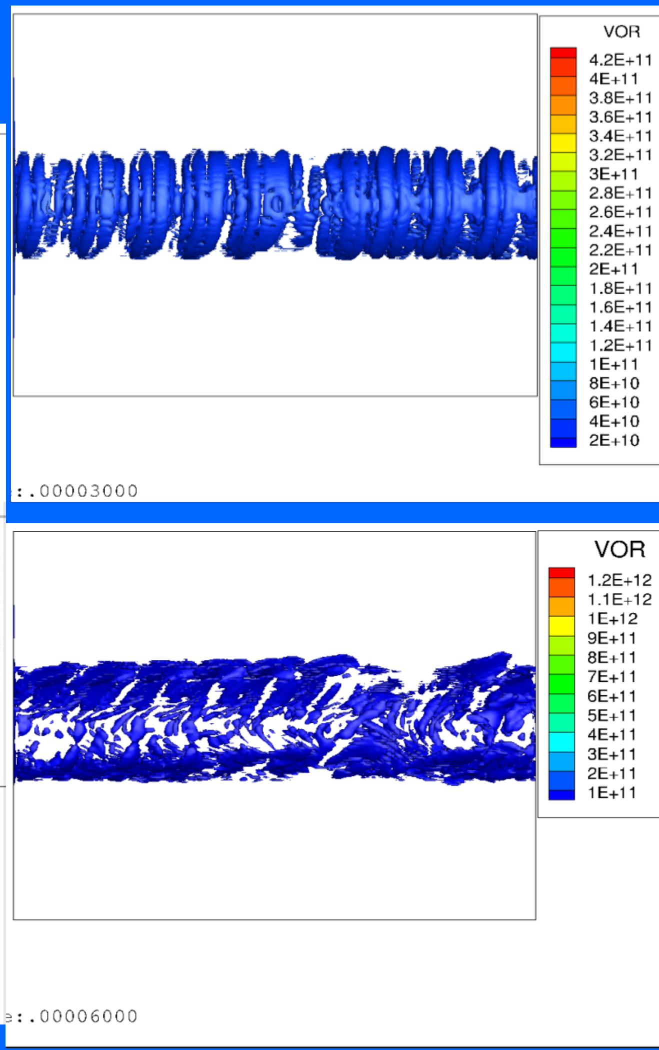


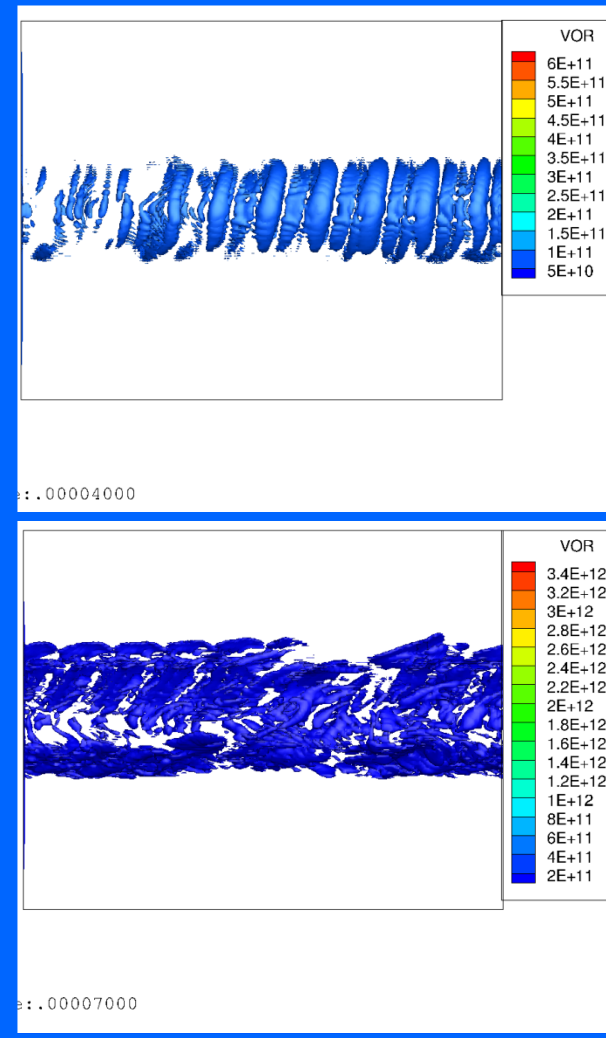
Vorticity

Originally axisymmetric; rings tilt; streamwise vorticity develops.
Vorticity magnitude increases with time.
Maximum vorticity in braid region.

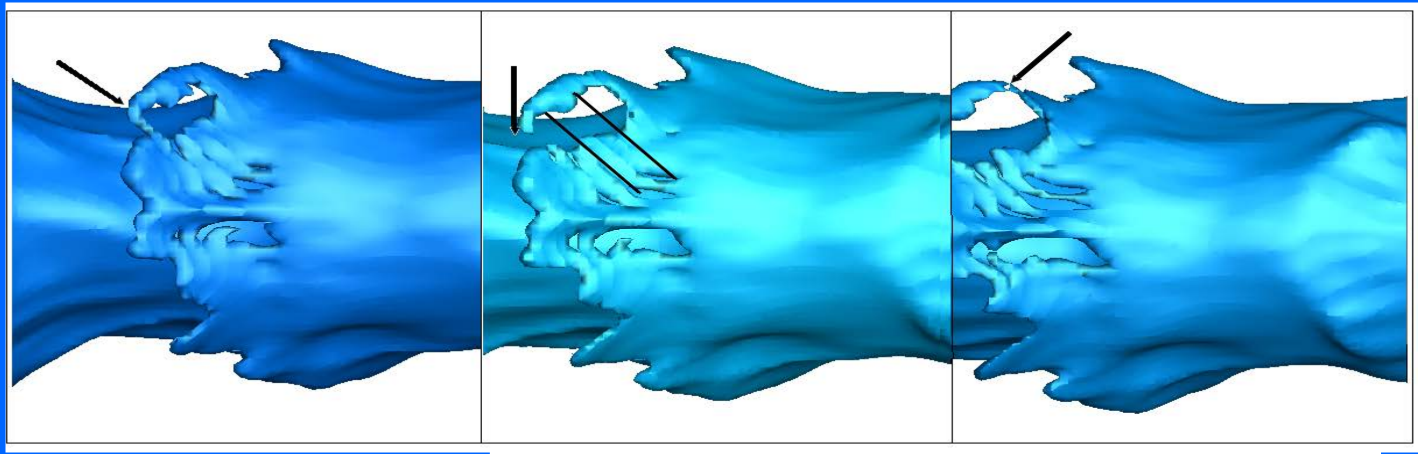
Re = 1600
We = 230,000
$\mu_1 / \mu_2 = 9 \times 10^{-4}$
$t = 30\text{-}70\ \mu s$



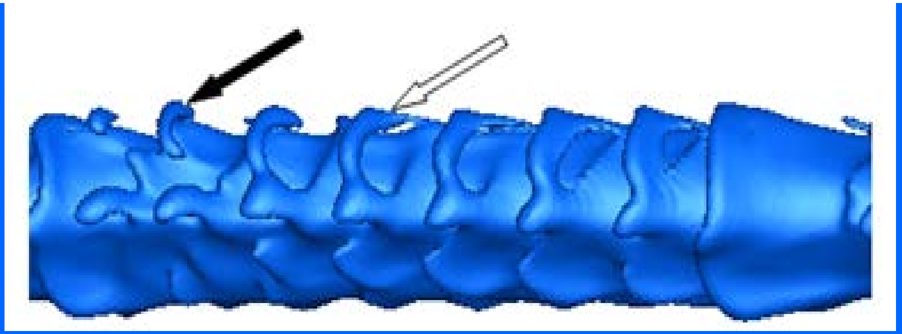




Lobe and ligaments   *time* →



- *We* = *230000*
- *Re* = *1600*
- $\rho_1 / \rho_2 = 0.1$
- $\mu_1 / \mu_2 = 9x10^{-4}$

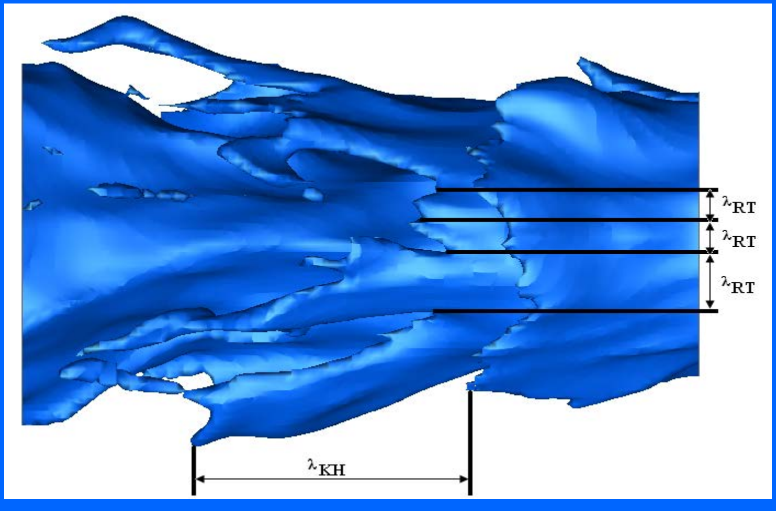


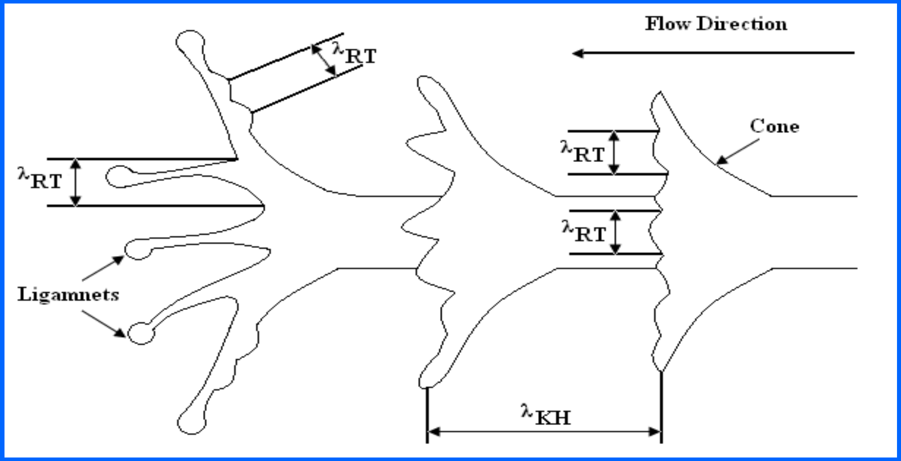
- *We* = *23000, Re* = *1600* ,
- $\rho_1 / \rho_2 = 0.1$, $\mu_1 / \mu_2 = 9x10^{-4}$

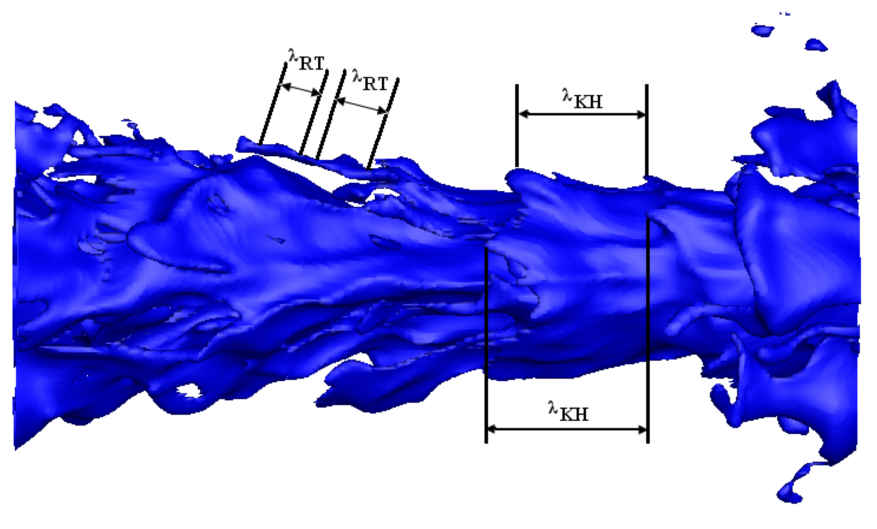
- **The cone-shaped liquid surface is sheared at the corners of the lobe where the maximum vortical motion occurs.**
- **The stretching thins the lobes yielding holes.**
- **The detached liquid from opened hole produces two ligaments that stretch away from core into the gas.**
- **The capillary effects can break the ligaments into droplets.**

Wavelength Analysis on Unstable Liquid Jet

K-H waves form cones, azimuthal R-T waves form on cones, ligaments result from R-T waves, capillary action plus more R-T waves on ligaments.



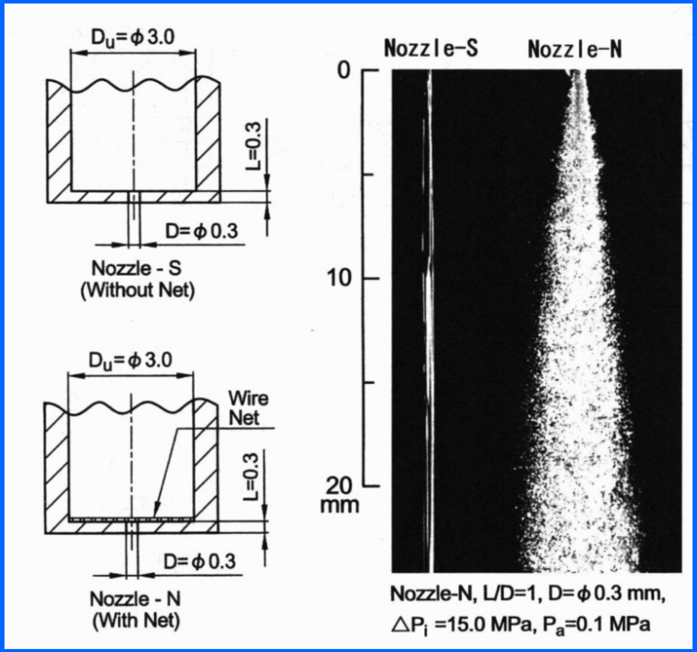




- **Transverse R-T instability wavelengths on the crests varies between 20 to 70 µm.**
- **R-T instability wavelengths on the ligaments vary between 15-50 µm.**
- **Acceleration is the main factor to develop the R-T instability on the ligaments and crests .**

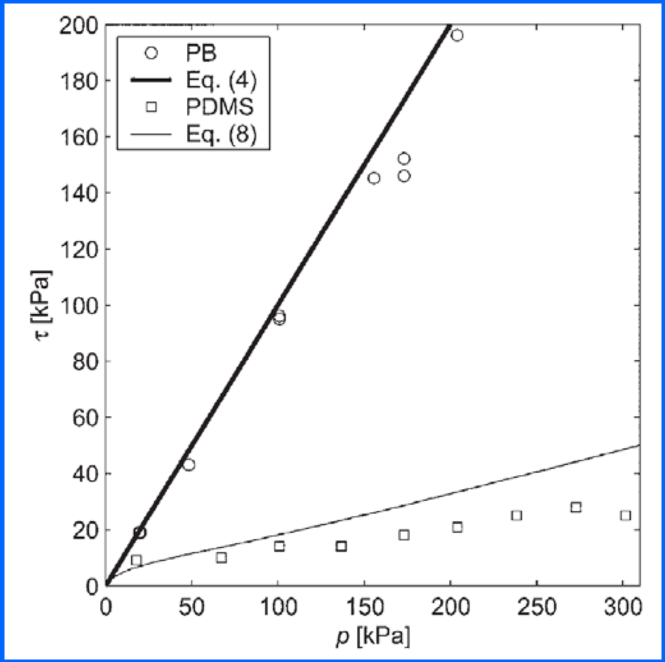
Cavitation in high pressure injectors

Cavitation occurs well above classical threshold pressure and has huge impact on jet breakup.



← Hiroyasu and co-workers (1998, 2000, 2001)

Kottke, Bair and Winer (2005) →



- - Total-stress criterion proposed by Winer and Bair (1987) and independently by Joseph (2001):

$$T_{11} > -p_c$$

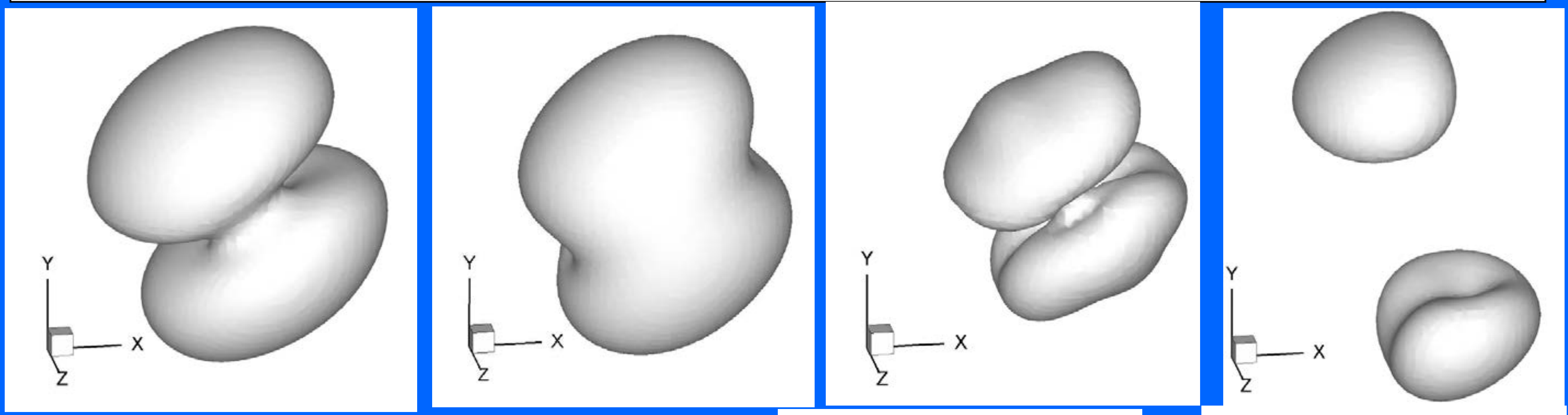
$$\mathbf{T} = \mu\left[(\nabla\mathbf{u}) + (\nabla\mathbf{u})^T\right] - p\mathbf{I}$$

(Maximum principal stress)

Calculation of bubble dynamics and far-field impact in strained flow

Dabiri, Sirignano, and Joseph (2007, 2008, 2010a, 2010b)- NS /LS

Bubble shape after rebound. From left to right: Case 1, Re=1.0, We=0.1; Case 2, Re=0.5, We=0.1; Case 3, Re=1.0, We=∞; Case 4, Re=1.242, We=0.0298.



In some cases with greater pressure variation, holes were formed upon collapse; threads developed connecting two masses and then broke.

Summary

Stretching of liquid sheets can result in tearing and hole formation.

Vorticity dynamics explain liquid jet deformation. We can learn from experiences with single-phase jets and should seek to unify the single-phase and multi-phase phenomena.

Density differences and surface tension modify vorticity.
R-T instability is best explained through vorticity dynamics.

Variation in pressure can cause changes in capillary instability and vorticity dynamics. Still, cavitation is possible at high injector pressures.

THANK YOU.